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Research Relative to Automated Multisensor Image Registration

Final Report on

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ABSTRACT

The basic approaches to image registration are surveyed. Three image models are presented as models of the subpixel problem. A variety of approaches to the analysis of subpixel analysis are presented using these models.

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PREFACE

This report summarizes studies conducted on Research relative to Automated Multisensor Image Registration, during the period April 1981 - April 1982 by Dr. Laveen N. Kanal (Principal Investigator), Mr. David Lavine, Mr. Charles Herman and Dr. Eric Slud. This work was supported in part by NASA Grant NAG5-154 to the University of Maryland and in part by L.N.K. Corporation, Silver Spring, MD. The main purpose of these studies was to review current approaches to image registration relevant to Landsat images with emphasis on feature matching and subpixel accuracy, and to recommend research which should be pursued in the area of registration error and subpixel accuracy analysis and estimation.

Dr. H.K. Ramapriyan, Information Extraction Branch, Code 932, NASA Goddard Space Flight Center, Greenbelt, MD served as technical officer for the NASA grant. The principal investigator is grateful to Dr. Ramapriyan for helpful discussion and for the references provided by Dr. Ramapriyan on Landsat image processing procedures.

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I. Introduction and Summary

This report deals with two aspects of image registration encountered in the processing of Landsat images: feature matching and sub-pixel accuracy. Many recent research efforts in the area of registration have considered the need to transform images into a form more suitable for accurate registration. These transformations range from simple linear filters to complicated procedures based on knowledge based image segmentation. The design of optimal transformations is difficult due to the wide range of types of transformations under consideration.

Certain classes of transformations yield highly simplified images which may be registered either as rectangular arrays of pixels or as sets of features. Examples of such simplified images are binary edge and sparse point images. Registration of edge and point images appears to be robust in comparison with many grey-level correlation methods. Edge and point image registration are important. In Section II we briefly review basic methods in this area. We also include a description of work we have done with one type of feature matching procedure.

Section III is a brief review of work on estimation of the errors inherent in image matching. This material is particularly important, since large errors in a matching scheme preclude the possibility of extracting subpixel information from that scheme. Included in this section is a discussion of a paper describing the selection of control points to minimize matching errors.

Analytical studies in the area of sub-pixel accuracy have been limited. In Section IV we give an overview of some basic ideas in the area. Though the analysis of this section is limited, we feel that a more detailed analysis of these model situations is feasible.

Section V contains a description of the research we are recommending on edge-based image matching and the analysis of subpixel accuracy. Three distinct approaches are described, ranging from a selection of "best" edge features to a theoretical analysis of the maximum accuracy attainable in a mathematical model of the image. The last section, the bibliography, includes

some items not cited in the body of the report. For the sake of completeness, we have included many articles on image matching which have indirect bearing on feature matching or sub-pixel accuracy since they do contain potentially usefull information on filtering and image modeling.

II. Feature Matching Review

II.1 Abstract Vector Matching

The recent development of various registration algorithms employing some type of feature matching emphasizes the need for more detailed analysis of the capabilities of feature matching. In this section we describe a feature matching method, called the Abstract Vector Matching Method (AV), which has performed well in simulation as well as experimentation with aerial imagery. The Landsat registration accuracy requirements require high accuracy on a large percentage of the images processed. To ensure this, a registration method must be insensitive to many sources of grey-level variation such as crop growth, soil moisture and very local scene changes. Feature matching methods offer considerable promise in providing this robustness. A variety of studies performed by L.N.K. Corporation have led us to believe that the A.V. method captures many of the desirable properties of other feature matching methods while incorporating some new approaches to the problem.

The basic A.V. method can be stated briefly. First, find a set of points of interest in the image, such as road intersections or points of high curvature on curves such as rivers. Second, select certain pairs of those points to be joined by vectors. The vectors may be real image edges or arbitrary pairings of points, which we call abstract vectors. Third, carry out the same procedure on a map or a control image. Fourth, for each vector in one image, determine which vector in the other image could align with it on the basis of length and possibly other factors such as image intensity on the two sides of the vectors. Each possible matching pair leads to an affine transformation taking one image to the other. Such a transformation is given by a set of rotation, translation, and possible scale factors. The parameters defining a transformation may be thought of as a point in Euclidean space. Clustering is performed in this space on the set of transformations

corresponding to matching vectors. The cluster centers are regarded as potential registration transformations. Each such transformation is then applied to the vectors in the sensed image and a figure of matching merit is computed. The transformation with the highest merit is taken as the correct transformation.

Two features of this algorithm deserve further comment. First, the selection of intersections and abstract vectors adds robustness to the detection process. Matching procedures based upon edges perform poorly if the edges become fragmented during the detection process, since the ends of the edges become difficult to locate. In spite of this fragmentation, the intersection of edge segments is often preserved. While matching could be performed directly on the set of intersections, the matching could be performed faster by imposing additional structure on the point set. Selecting pairs of points, joining them by vectors and matching vectors is considerably faster than investigating all possible matchings of points. Since intersections may be reliable even when edge endpoints are not, we do not require pairs of endpoints to correspond to endpoints of a real edge.

The second feature of interest in the algorithm is the clustering of transformations. Each matching of a vector in the sensed image with a vector in the reference image gives rise to a feasible registration transformation. Clustering transformations corresponds to finding those transformations approximately maximizing the number of vectors matched. Clustering enables the algorithm to avoid examination of many incorrect transformations.

II.2 The Rockwell Pattern Matcher

An image registration algorithm developed at Rockwell International Corporation is described in [Belsher et al. 1979]. The technique is based on matching high contrast edges. Experiments have been performed in both the infrared and optical portions of the spectrum. Since the pattern matcher was not tested on Landsat data, it is difficult to extrapolate the numerical results to Landsat registration accuracy. In spite of this, the methods of analysis and the experimental results give some insight into the possible benefits of subpixel interpolation.

The feature extraction algorithms were designed to maximize the peak to sidelobe ratio, defined as the number of matching features at the correct offset divided by the maximum number over all incorrect offsets. In order to carry out this maximization the probability of the existence of a reference feature given a sensed feature was determined and maximized for each feature extraction algorithm. The expected number of matching features at the correct offset is:

$$N \ \mathrm{Prob}(S \cap R)$$

where $\mathrm{Prob}(S \cap R)$ is the probabilty of a match between a sensed (S) and corresponding reference (R) feature at the correct offset, and N is the number of features in the reference. At incorrect offsets, sensed features are independent of corresponding reference features. Thus the expected match at incorrect offsets is given by:

$$N \ \mathrm{Prob}(S)\mathrm{Prob}(R)$$

where Prob(S) and Prob(R) are the sensed and reference feature densities. Thus the ratio of the expected peak to the expected sidelobe is

$$(P/S) = \frac{\mathrm{Prob}(S \cap R)}{\mathrm{Prob}(S)\mathrm{Prob}(R)} = \frac{1}{\mathrm{Prob}(R)} \ \mathrm{Prob}(R/S)$$

where Prob(R/S) is the probability of a reference feature given a sensed feature. The reason for choosing the feature extractor maximizing P(R/S) is that Prob(R) can be computed from the reference image.

Edge features were obtained by median filtering to remove noise spikes, followed by the application of a gradient operator. Further processing is done to remove the effects of local variations in grey scale density, and finally a template extractor is used to obtain the endpoints. Spot features are also extracted.

Each feature matching leads to a possible matching of a reference to a sensed image. For N such possible matching we denote the a posteriori probability of offset i being correct given the sensed image by $p(i/S)$, $i=1,\ldots,N$. Using Bayes theorem it is seen that

$$p(i/S) = \frac{p(i)p(S/i)}{p(S)}$$

In general, flight information can be used to estimate $p(i)$, and $p(S)$ is common to the evaluation of all offsets, so it suffices to compute $p(S/i)$.

The sensed features, S_j, at the correct offset are assumed to take the form

$$S_j = R_j + N_j$$

where R_j is the reference measurment and N is noise. The measured features are mostly lengths. From the above decomposition of S_j, the following expressions for $p(S/i)$ results:

$$p(S/i) = p(\overline{N}) = p(\overline{S} - \overline{R})$$

where $\overline{N}, \overline{S}$, and $\overline{R}$ are vectors with coefficients a_j, S_j and R_j respectively. Finally, assuming the errors are independent the following expression for $p(i/S)$ may be derived:

$$\log[p(i/S)] = \log p(i) + \sum_j \log p(S_j - R_j)$$

Some modifications are required to handle problems with very small probabilities. A distributional form for p must be assumed with reasonable choices including the Gaussian, Laplacian and Cauchy densities.

We may think of $p(i/S)$ as a correlation surface which is a function of offsets i. The surface is approximated by a

quadratic surface using interpolation on a 3x3 pixel area or a 5x5 pixel area. Registration was performed on 3-D imagery using fifteen images. The root-mean-square registration error with no interpolation was 2.3 pixels. Using subpixel interpolation the average error was reduced to 1.9 pixels. In only one case was the error increased using subpixel interpolation.

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II.7 Hybrid Correlation

The trade-offs between correlation and feature matching have not been studied adequately. Ratkovic [1979] outlines basic issues in assessing this trade-off and describes a hybrid correlator combining desirable features of both methods. Feature definitions are given and the relevance of feature characteristics to correlation performance are described.

Accuracy, as described in Ratkovic's work, refers to the width around the true correlation peak in which a correlation point is likely to lie, rather than the accuracy attainable through interpolation of the correlation function. Three basic issues are discussed. How does one quantify accuracy? What factors affect it? What methods improve it? It is assumed that a two stage matching process is available in which the first stage determines an approximate match, while the second stage improves local accuracy of the registration.

Based on limited experiments, it appears that the three primary factors affecting registration accuracy are the number, size, and mean intensity of homogeneous regions in the image. A homogeneous region is defined to be a set of spatially connected pixels which posses the property of at least first-order stationarity and possible second-order stationarity. Several experiments were conducted to determine the significance of homogeneous regions. First, a 20 x 20 reference scene was generated using a Gaussian distribution with zero mean and variance one. The center 10 x 10 image was designated a sensor scene and correlated against the reference scene. To introduce the effect of nonhomogeneity, a bias value was added to all values in the left half of the scene. Correlations were performed for offsets μ = 0,1,10,100. As the bias level increased, the correlation function became progressively flatter near its peak.

Further experiments were conducted with ERTS satellite data from several types of areas (agricultural, mountain, suburban, and desert). The images were divided into homogeneous regions and correlations were performed in two ways. First the ordinary

correlation of the sensor and reference image was computed. Next, each pixel in a homogeneous region was replaced by the mean value of pixel intensity in the region, and correlation was performed. Little difference was observed using the two types of correlations. From the above experiments, as well as others, Ratkovic infers that the presence of large homogeneous regions of significantly varying mean intensities are the major source of peak spreading in the correlation function.

A "pure" feature matching algorithm was employed using edge detectors to determine region boundaries. Intersection points of region boundaries were then extracted together and the number of lines meeting at such intersections was recorded. A matching procedure was then applied to the vertices, weighted by the number of intersecting lines to determine the registration transformation.

A feature matching correlation procedure was developed. The images were decomposed into homogeneous regions and each region was independently normalized to zero mean and the resulting images were correlated. The primary difference between this method and the "pure" feature matching method is that in the former, region matches are weighted by their size, while they are not in the latter.

The final matching algorithm proposed by Ratkovic is a hybrid between correlation and feature matching. In this algorithm the reference image is segmented into homogeneous regions. At each displacement between the reference and the sensed image, the sensed image is segmented identically to the reference image, and corresponding regions are normalized and correlated. The correlations in the individual regions are added and the registration transformation is computed from this correlation function. The hybrid correlation algorithm avoids the difficulties of reliable feature extraction in the sensor image, while retaining the advantage of correcting for homogeneous regions.

Feature matching and the hybrid algorithm both essentially high-pass filter the image to remove the effects of varying mean

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intensities between regions. This high-pass filtering, which has received considerable attention in recent years, is still not well understood with respect to its role in the correlation process.

A conclusion of the Ratkovic study was that, in the presence of purely local noise, ordinary correlation performed best among the algorithms. For regional noise, such as a bias value added to an entire region, resulting from such factors as soil moisture or crop development, the pure feature extraction method performed best. The hybrid correlator gave somewhat less accurate registration information than the pure feature extractor, but at less cost.

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III. Error Estimation Review

III.1 Image Correlation with Geometric Distortion

The estimation of translation parameters in image registration is a special case of the image registration with an affine distortion. Local registration and false acquisition probabilities in this more general case were studied by Mostafavi and Smith [1978]. They determined the optimal reference image shape and size corresponding to a given rotation and scaling between the reference and the sense image. The rotation and scaling parameters are determined, prior to registration, using knowledge of the sensor position and attitude. In the presence of rotation and scaling changes, two reference window properties must be balanced. First the window should be large enough to obtain a reliable estimate of the registration parameters. Second, the window should be small enough to prevent the geometric distortion from degrading registration quality, which can be caused by poor alignment of pixels that are far removed from the center of rotation.

Mostafavi and Smith draw further conclusions about the structure of ideal reference images. For a fixed rotation and scaling change, they show there is an ideal reference image size and shape which minimized the probability of selecting the wrong lobe in the correlation function. There is an ideal reference image size and shape which minimizes the error in registration, given that the correct lobe is selected. The ideal window size in the latter case is smaller than the ideal window size in the former case. Both the probability of false acquisition and the local registration error depend on the difference matrix I-A where I denotes the identity matrix and A defines a rotation and scale change. Before discussing the significance of these conclusions, we present the assumptions of the model.

The reference image, I, (x) = P(x), where $x = (x_1, x_2)$ is a function of two variables where the origin is assumed to be the reference point. The sensed images, I, (x) is given by

$$I_s(x) = P_d(x) + N(x)$$

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where $P_d(x)$ is a geometrically distorted version of the reference image and $N(x)$ is additive noise. It is assumed that $P(x)$ and $N(x)$ are uncorrelated, zero-mean Gaussian random processes which have spatially shift-invariant, second-order statistics. In addition the cross-correlation is assumed to be Gaussian. The patterns are assumed to be differentiable in the mean-square sense. The geometric distortion is given by

$$P_d(x) = P(A_x + t_0)$$

where A represents a rotation and a scale change and t_0 is a translation. The basic measure of correlation acquisition accuracy is the expected value of the correlation process of the true translation divided by the standard deviation of the correlation process far from this point. The local registration accuracy is a function of the gradient of the correlation function at t_0. For a correct registration, this gradient should be zero.

A basic difficulty in applying this work lies in the assumptions made in the analysis. The expected value of the correlation function at t_0, and the standard deviation of the correlation far from t_0 are computed independently. Thus a more realistic but less tractable problem is to look at the value of $C(t_0)$ and $C_1 = \{ \max C(t) \mid t \text{ far from } t_0 \}$ for each realization of the Gaussian picture process and determine the probability of the event $C_1 > C(t_0)$. Similarly, the value of the gradient of $C(t)$ as a measure of registration accuracy is difficult to relate directly to the expected registration error in pixels.

III.2 Image Registration Error Variance

The variance of the translation error in registration is one measure of registration performance. Two procedures for estimating this variance [McGillem and Svedlow 197] will be described in this section. While the variances compared using these methods may be less than a pixel, the authors assume that the log likelihood function for the image to image displacement can be modelled by a second order polynomial. This assumption does not appear to be derivable from more fundamental properties of their model, nor is any experimental evidence for its plausibility provided. As a result of this assumption, the role of various correlation function interpolation schemes in registration accuracy is bypassed, thus limiting the effectiveness of these models for error variance. Though the relevance of these variance estimation schemes is currently limited by the quadratic model, the basic ideas of these methods will be presented due to the possibility of future improvements on the method.

In both methods, it is assumed a model for the underlying image is known. Method 1) makes the following additional assumptions:

1) The sensed image consists of additive noise which is independent of the model.
2) The joint probability density function of the noise is Gaussian.
3) The a priori distribution of the translation parameters is uniform over the range of interest.
4) The variance in the x and y directions may be modelled separately.
5) The final result is dependent upon a large signal to noise ratio.

The Likelihood function of the displacement parameters is given

by

$$\Lambda(T_x, T_y) = p_m(T_x, T_y) \frac{p_m(T_x, T_y)(f)}{p_0(f)}$$

where

$\Lambda(T_x, T_y)$ = Likelihood function of the the displacement parameters T_x and T_y

$p_m(T_x, T_y)$ = density function of the parameters. T_x and T_y, given the known signal

$p_m(T_x, T_y)(f)$ = conditional density of $f(x,y)$ given $m(x,y,T_x,T_y)$ is present

$p_0(f)$ = conditional density of the $f(x,y)$ given $m(x,y,T_x,T_y)$ is absent

$m(x,y,T_x,T_y)$ = known signal as a function of the spatial coordinates and the displacement parameters

$F(x,y) = m(x,y) + n(x,y)$ = received signal

$n(x,y)$ = additive noise

The variance of the error in the x direction translation will now be estimated. Denoting the estimates of T_x and T_y by $\hat{T}_x$ and $\hat{T}_y$, first expand the log-likelihood in a Taylor series about T_x, and then use the fact that at a maximum of the likelihood function, the first order derivatives are zero. The following expression is obtained:

$$\ln \Lambda(T_x, \hat{T}_y) \approx \ln \Lambda(\hat{T}_x, \hat{T}_y) + 1/2 \frac{\partial^2 \ln \Lambda(\hat{T}_x, \hat{T}_y)}{\partial T_x^2} (T_x - \hat{T}_y)$$

The variance in the x direction is then given by

$$\Delta_x^2 = - [\frac{\partial^2 \ln \Lambda(\hat{T}_x, \hat{T}_y)}{\partial T_x^2}]^{-1}$$

Expanding $\Lambda(T_x, T_y)$ it can be shown that

$$\frac{1}{\Delta_x^2} = \sum_g \sum_h Q_{gh} \frac{\partial m_g(\hat{T}_x, \hat{T}_y)}{\partial T_x} \frac{\partial m_h(\hat{T}_x, \hat{T}_y)}{\partial T_x}$$

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Here, the Q_{gh} are terms in the inverse of the covariance matrix of the noise, g and h represent index pairs $g=(x_i,y_j)$, $h=(x_k,y_l)$, and the sum is taken over all indices. The above expression can be simplified to yield a variance estimate in terms of the effective bandwidth and signal to noise ratios in the x and y directions.

The second method assumes the noise is additive and independent of the signal; the noise spectrum is known and a matched filter is used. The basic model assumes the received signal is a sum of two output signals; the model convolved with a filter and the noise convolved with the same filter. First the functions representing the filtered model and the composite filtered output signal are expanded in Taylor series about the true registration position. It is assumed that the filtered outputs are maximized at the true registration position. Applying the corresponding constraints on the derivatives, an expression for the error variance is obtained in terms of the filtered output noise and the filtered model output. By selecting a matched filter, a variance estimate similar to that of method one is obtained.

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III.3 Control Point Selection

The spatial distribution of ground control points (GCP) can affect the accuracy of image registration. Clearly the number and location of GCP's is constrained by the scene under consideration, but subject to this constraint, a registration program must still select a set of ground control points. This selection procedure should choose control points so as to minimize the registration error. In a study of this problem, Orti [1981] developed an expression for the mean square Landsat MSS registration error as a function of GCP locations. A minimization procedure is then applied to determine the optimal GCP distribution.

Orti's procedure estimates the attitude and altitude of the satellite from a set of GCP's. All other mapping functions relating the raw and corrected images are assumed to be known. In addition it is assumed that within a Landsat frame the three attitude angles can be adequately described by cubic polynomials of time and the altitude by a linear function. Thus there are fourteen coefficients to be estimated. The error criterion is the average error in these coefficients.

The geometry of the transformation problem will now be described. All deviations are assumed to be small. The attitude and altitude deviations from the nominal values are given by

$$x = \phi + k\tan G(p)$$
$$y = w[1 + \tan^2 G(p)] + h\tan G(p)$$

where p is the GCP's pixel number within the scan line measured from its center, G(p) is the corresponding scan angle, zero at the center of the scan line; ϕ, n, and k are the attitude angles, pitch, roll, and yaw and h is the relative altitude deviation, all measured at the time when the GCP was seen by the satellite. Here x and y represent the GCP control differences in position along and across the orbital track on a plane tangent to the earth through the nadir point of the image. The x and y coordinates represent differences in positions between the GCP in

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map coordinates and the GCP position obtained from transforming the raw image coordinates without consideration of attitude or altitude deviations from the nominal values.

The attitude angles are then approximated by third degree polynomials using orthonormal Legendre polynomials and the altitude is approximated by a first degree polynomial. The expressions for x and y together with the Legendre polynomial expansions yield the following matrix equations for deviations (x_i, y_i); $(i=1,\ldots,n)$ corresponding to GCP's c_{x_i} and c_y.

$$X = W_x \cdot C_x + \varepsilon_x$$
$$Y = W_y \cdot C_y + \varepsilon_y$$

ε_x and ε_y are vectors representing the errors associated with the control points. W_x and W_y are matrix functions of F and the Legendre polynomials entering in the expansions. From the above matrix equations, least-mean-square estimation of c_x and c_y can be computed. These estimates can then be used to compute the variance of the propagated registration error. After a somewhat involved derivation of the registration error as a function of GCP location it is determined that GCP's should lie near the four corners of the image and in some areas of the left and right edges of the image where the positions are a function of the number of control points.

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IV. Sub-pixel Accuracy Review

IV.1 Sub-pixel Edge Estimation

Three approaches to the estimation of edge locations to sub-pixel accuracy are presented in a recent paper of Hyde and Davis [1982]. In two of the approaches, the image is assumed to be a "grey level" image in which the intensity at each image point can take on a range of values. A set of pixels is assumed to have been identified as belonging to an edge which is assumed to separate two regions. The regions are assumed to have either constant, but different, grey levels, or grey levels given by two different but known distributions.

For each pixel and each possible line passing through the pixel, the area on each side of the line is computed. The grey level of an edge pixel is assumed to be a weighted linear combination of the random variables giving the grey levels in the two regions. The weighting is by the relative area of the pixel belonging to each region. The edge pixels' grey levels are assumed to be independent. If we parameterize the edges by polar coordinate (ρ,Θ), then the probability density function for the observed grey level in an edge pixel can be calculated. This density is parameterized by (ρ,Θ), since we get a different grey level mixture depending on how the edge intersects the pixel.

The estimation of the edge parameters, (ρ,Θ), can now be set up as a maximum likelihood or least squares procedure. This is straightforward to set up since the intensities are independent random variables with known densities. The likelihood estimate is difficult to compute numerically and no tests were performed using this approach.

A simpler approach to the problem was also investigated. The edge was estimated to be the best least squares fit to the centers of the edge pixels. No use was made of grey levels in this approach, and no optimality was claimed. A refinement of all the above methods was also studied. In this refinement, edges were fit to each of three consecutive points on a line, rather than globally fitting as described above.

The methods were tested on ten Landsat images in which the edges consisted of agricultural field boundaries. The edge pixels were selected using ground truth rather than an edge detector, as would be necessary in practice. Bands 1 and 4 were selected, since they tend to be different and independent. Results are reported for least squares fitting using grey levels and least squares fitting using pixel centers. Both local (three point) and global fitting experiments were performed. For global fits, ground truth and estimation results for p and were given. For local fits the rms errors in Θ and p were reported. The rms errors in p were less than one in all cases and the rms errors in Θ (in degrees) ranged from about .3 to 12, with results clustering about 1.5. The fitting to pixel centers was as accurate as the edge fitting using grey levels. Thus the use of grey levels did not appear advisable, due to the additional computational complexity.

The experimental results of this work are difficult to assess because the accuracy is given in terms of p and Θ rather than in terms of pixels. Global measures of registration accuracy are not readily computable from the given data. This is not a criticism of their methods, but rather indicates the need for further work before their study can be adequately assessed. No analysis of the probabilistic properties of the estimated registration transformations are given in this work. We believe that an analysis of this type is feasible for methods similar to those used in this study. Such an analysis could provide a useful basis for the evaluation of algorithms for registration with subpixel accuracy.

IV.2 Current Sub-pixel Methods

Two basic approaches to sub-pixel registration accuracy are discussed in a recent survey [Wolfe and Juday 1982] on registration. In the first approach, the correlation between two images is interpolated and its maximum is taken to be the offset. In the second approach, patches in the two images are matched to the nearest pixel using correlation, and a mapping function is fit to these matches. The success of this method is based on the assumption that the errors in local patch fits are random and tend to cancel.

Various approaches have been taken to the interpolation of the correlation function. Fitting of lower order polynomials to the correlation function is one common method. Both bivariate and fourth-order polynomials have been used for this purpose. Fitting is commonly done over a small neighborhood, up to 5x5. The order of the polynomial can be allowed to vary. The quality of the peak in an interpolated correlation function can be evaluated in terms of the curvature at the peak and the height at the peak.

Several variations to the above approaches have been tried. Bivariate gaussians rather than polynomials have been used for fitting; elliptical cones have also been studied. The centroid of the correlation surface has also been considered as an estimate of image offset. As a final approach, the image offset can be estimated by the phase function in the correlation transform. The offset can be directly calculated from the phase if the correlation function is symmetric, which unfortunately may not be the case.

The above methods are not based on any theoretical model of the correlation process. As a result, the accuracy of interpolation depends largely on the surface fitted to the correlation. Experimental studies do not provide an adequate basis for a clear comparison of these methods at this time.

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V. Research Recommendations

V.1 Geometric Registration

Interest in performing image registration using feature matching methods is growing in the image processing community. The temporal stability of edge locations is considerably better than that of grey levels for a wide variety of scenes. Correlation of images filtered for edge enhancement may be regarded as an intermediate step between traditional correlation methods and more symbolic feature matching. In this section we examine some issues in edge based image matching. We think work along these directions could shed considerable light on the nature of sub-pixel accuracy and the design and evaluation of algorithms to achieve it.

The study of sub-pixel accuracy in registration may profitably be divided into two areas of research. First, how much information about sub-pixel accuracy is contained in a pair of images. Second, how algorithms can be designed to achieve sub-pixel accuracy. In part A of this section we deal with the first topic, and in part B with the second.

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V.1.A Information Content of Edge Pixels

The information content of edge pixels is of considerable importance for several reasons. Knowledge of the maximum registration accuracy attainable gives a benchmark by which to evaluate algorithms, it can reveal situations in which it is fruitless to pursue the sub-pixel problem, and it may give rise to more accurate registration algorithms.

The design of models for feature based matching is at a much more primitive level than is the modeling of grey scale images. A variety of random field models are currently in use in the study of grey scale images. Though there is considerable question as to the usefulness of these models, they have proved useful as a study point in the analysis of the reigstration process.

We confine our modeling discussion to binary edge images. Various arguments can be made for and against such a restriction. On the positive side, such images are easily attainable through a variety of edge detection techniques. The simplicity of the image greatly reduces the information content of the picture, thus allowing more extensive processing to match images. This simplicity also offers hope that the situation can be analyzed. On the negative side, the relationship between the binary image and the original image is no longer clear. Unless reliable information on the accuracy of edge detectors is available, it is difficult to evaluate the accuracy of the binary image registration.

The objection raised above to the use of binary images for registration can be softened by several factors present in the Landsat context. Registration procedures currently used with Landsat images can register to within a pixel on a large number of scenes. Current attempts at sub-pixel accuracy separate the registration procedure into rough and fine registration, where rough registration is done by correlation and fine registration is done by interpolation. A similar approach can be adopted with geometric matching.

The expense of reliable edge detection can be reduced

considerably if prior knowledge of approximate edge location is available. To use such knowledge, a rough registration is necessary. One approach to registration along these lines is to perform a rough registration using correlation, and an edge database to perform edge location in a small number of areas where edges are known to be present. Knowledge of edge properties, such as spectral signature, together with the edge location information available from the rough registration, could greatly enhance edge accuracy. Edge points such as roads could also be sought. In this constrained edge searching environment, estimation of edge location accuracy may be plausible.

The prior edge knowledge allows us to restrict ourselves to a very small number of reliable edges. In this section we restrict ourselves to a single edge or a pair of parallel straight edges. This restriction is imposed because it is the natural first step in the analysis of binary edge images. In practice, this condition is enforceable by throwing away all other edges.

We now come to the fundamental question of this section. Given a set of pixels representing the digitization of an edge, to what extent is the exact location of the edge determined? This question requires some amplification before it can be formulated precisely. Do we view a real world edge as having thickness or as an ideal line segment? Which pixels represent an edge? If an ideal line segment intersects a pixel very near a corner, should that pixel indicate an edge point? Should false edge points be allowed? Though these considerations are somewhat obvious, the specific model adopted can greatly affect the course of analysis.

We first consider the noise free model. In this case, each pixel which is set to one must contain at least part of an edge. All other pixels are set to zero. Note that we have not said that each pixel which contains part of an edge must be set to one, only its converse. Thus we allow for the possibility that our sampling mechanism requires that an edge be at least a certain distance into a pixel before the pixel is set to one.

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As a first step toward a more careful formulation of our basic question, we ask what it means for two line segments to be close. There are many definitions which could be used, but we must recall that whatever definition is adopted must be useful for registration. Viewing line segments as sets, the standard definition of distance between sets (minimum distance between pairs of points, one in each set), appears useless, because of the possible wide variation in line position among lines close in this sense. Similarly, viewing lines as parameterized by polar or cartesian coordinates, we can define closeness in terms of nearness of the corresponding parameters. This also can result in considerable separation between lines unless line segment lengths are explicitly considered.

Several feasible definitions of line segment closeness can be given. We now describe one appealing measure. Let L_1 and L_2 denote two continuous (not digital) line segments. For each pixel $x \in L_1$, let $d_1(x)$ denote the Euclidean distance from x to the nearest point in L_2. Similarly, for $y \in L_2$, let $d_2(y)$ denote the distance from y to the nearest point in L_1. Let

$$D_1 = \max_{x \in L_1} d_1(x) ,$$

$$D_2 = \max_{y \in L_2} d_2(y) ,$$

and $$D = \max (D_1, D_2) .$$

Then D is a measure of line closeness which guarantees that lines never get far apart in the sense that no point on either line is very far from the other line. For a pair of line segments given in cartesian coordinates, formulae for this measure can be easily derived.

Several possible definitions of closeness have been presented. Of these the last was characterized as feasible for registration. This statement requires some justification.

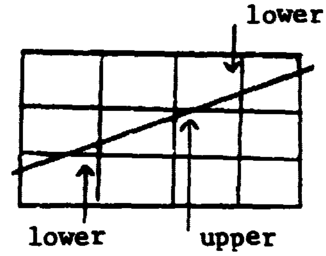
Suppose we have observed a set of pixels representing an edge. Assume furthermore that any two real edges, which could have caused this set of pixels to be set to one, can be proven to be close in the above sense. What can we say about the true location of the line in the image? Recall that the segment we are observing digitally is known to correspond to a straight segment, e.g., a section of road between two intersections.

We wish to determine a rigid mapping from the road segment into the image. Suppose we require the mapping to send the road segment center into the center of one of the possible lines going through the observed pixels. Suppose we further require the mapping to send a whole interval surrounding the road center to an interval surrounding the line segment center. Then it can be shown that if any two feasible lines are sufficiently close by our distance measure D, then the maximum registration error can be made less than a pixel and in fact the error can be bounded.

Even in the simple situation described above, the hard part of the task remains. Given a set of pixels how does one determine the maximum distance between feasible lines. A related question which is of vital importance in this area is the following. Is it true that for most line segments which could be placed on a sampling grid, the resulting set of pixels is rigid? By rigid, we mean that the set of feasible lines for that segment are close. A further question of greater practical importance is what length and directional properites of a line segment guarantee rigidity? Finally, how can a set of line segments be selected to guarantee rigidity if single segments are unreliable?

The answers to the above questions are not yet available, but we are currently investigating these questions under another research grant [NASA Subcontract L200081]. We briefly describe a method of attack which we consider to be fruitful. A line is rigid if slight changes in its location will result in a change in the set of pixels taking the value one. We are currently attempting to show that certain relations between the line length and the line angle guarantee rigidity. Roughly speaking, the conditions guarantee that the line runs near the corners of

several pixels. Furthermore, the pattern is approximately alternating in that the line goes near a lower corner of a pixel, then near an upper corner, then near a lower corner, etc. These corners may be separated by several pixels.



This condition can be formulated in terms of certain basic functions occuring in elementary number theory. The statement that the condition holds most of the time is related to some basic results in probabilistic number theory.

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V.1.8 Binary Edge Location

The sub-pixel edge location information present in a binary edge has been disucssed. We now consider the question of how to extract this information. The problem may be subdivided into two parts. First, for a given binary edge, how can the set of feasible candidates for the true edge be determined. Second, how should one candidate be selected from this set as an optimal estimate. The detailed study of these questions is currently underway as part of a NASA grant [NASA Subcontract L200081].

The determination of the set of feasible edge locations based on an observed binary edge is a complex geometric problem, even in the optimal case where all observed edge pixels really belong to an edge. In this case, one could roughly fit an edge to the pixels using some method such as least squares, and then perturb the edge till it passes through the desired pixels. If pixel errors are allowed, then the edge might be constrained to go through at least a certain percentage of the pixels.

Once a single feasible line is located, the line can be perturbed to estimate bounds on the edge location. The precise manner in which these perturbations should be done must be determined by a detailed geometric analysis of the problem, which is currently under way. If reliable bounds can be found on the variation in edge location, then several approaches to the selection of an optimal edge can be adopted. First, one can define a distribution on edge locations, and compute the expected edge location. Due to the lack of information as to a reasonable distribution, a second approach may be preferable. In this approach, the "most central" line is located. This line is that feasible line which has the smallest maximum distance to the other feasible lines. This can be made precise in several ways, but the computational procedure for locating this line will depend heavily on the geometry of the set of feasible lines.

The feasibility of these geometric methods cannot be assessed at this time. They offer promise because they are based on a model of the relationship between binary images and edge locations. The major experimental study necessary to tune the

model is an analysis of the relationship between observed edge pixels and real edges. The primary quantity to be estimated is the percentage of detected edge pixels which are true edge pixels. This quantity might be estimated as a function of edge angle, length and content (road, field, etc.).

The study of edge location could be extended to more complex figures such as intersections, or strips such as wide roads.

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V.2 Analysis of Subpixel Algorithms

A. Background

In analyzing the problem of attaining sub-pixel accuracy, we must first ask whether, under general conditions, sub-pixel accuracy is achievable with a reasonable degree of confidence; a negative answer renders all other questions moot. Assuming a positive answer, the analysis can be divided into two parts: a study of existing algorithms, and the development of new algorithms which satisfy certain conditions of optimality.

In performing the above analysis, one of three (not necessarily disjoint) methods can be applied. The first is a trial-and-error-by-experimentation approach, in which "real" data are used for the experiments. This method is the least satisfying, since without a deeper understanding of the problem (which is exemplified by a model) real analysis is hardly possible. However, it is reasonable to test algorithms on such real data.

In the second method, a model is hypothesized. The consequences of the model are derived via simulations. An important advantage of this method is that relatively complicated models can be studied.

The third method also involves a model, however this time all results are derived analytically. To guarantee tractability, only simple models can be considered. Sometimes the model must be simplified to the extent that the very qualities of the problem which lead to fundamental difficulties are assumed out of existence. But if one feels that the model does capture the essential features of the problem, then this analytic method is superior to the others. It can lead to deeper insights and superior algorithms.

The work currently being done is restricted to the last two methods, with an emphasis on the last one. The model used, and the analysis performed, are extensions of the work done by Novak [1981]. He considered the problem of finding an edge, of known shape, in a grey level picture. This is essentially the same as

matching a binary image to a grey level image. Novak assumed the pixel grey levels are statistically independent, and within the homogeneous regions separated by the edge, are identically distributed.

To determine the location of the edge within the pixel, Novak assumes that the edge pixels have the same distribution as the other pixels, but with a mean which is a convex combination of the means of the pixels in the two homogeneous regions. The weights reflect the percentage of the edge pixel residing within each homogeneous region. Using this model Novak finds the MLE of the weights, which determine the location of the edge.

Because this method requires a knowledge of the means within each homogeneous region - these means are unknown - Novak recommends fitting a biquadratic surface to the nine correlation points centered at the point with maximum correlation. The peak of this surface determines the sub-pixel match point.

B. Recommended work

Using computer simulations we recommend determining whether sub-pixel accuracy is attainable. Because this problem would be studied via simulations, rather than analytically, a complicated underlying model can be assumed. Rather than two homogeneous regions, we could allow many such regions, with grey levels varying linearly over each region. Edge pixels would be assumed to be convex combinations of the grey levels of the neighboring pixels, rather than simply their means. We would also allow statistical dependence of pixel grey values within regions, maintaining independence from region to region. Note that we are allowing only translation errors between the two images.

Assuming favorable results from the simulation experiments, we would proceed to analytically study current matching algorithms. The model used is similar to the one described above, the difference being that the covariance between pixels of a region will take a simpler form in the present case.

The algorithms would be analyzed asymptotically for four

reasons. First, asymptotics are easier to deal with, hence more complicated models can be treated. Second, images contain enough pixels to make asymptotics reasonably valid - although how many are "enough" should be looked into. Third, asymptotics are often independent of underlying distributions (e.g., the Central Limit Theorem), hence results are valid for a large class of distributions. And fourth, it is an axiom of statistics that estimates should behave properly in the limit, hence the results of matching algorithms should approach the correct point as the size of the image increases.

Lastly we would seek to devise new algorithms based on the insights gained in the above work. It is clear that in general biquadratic interpolation does not behave properly in the limit - in fact this remark probably applies to any polynomial surface. The appropriate interpolating surface should be determined by the autocorrelation structure of the image, and in fact should peak at the same point as the likelihood surface over the image. It should be kept in mind that implicitly we are assuming that we can only see the (continuous) correlation surface at a discrete number of points, and that a second order Taylor series expanded about the maximum point is valid over the entire 3x3 neighborhood of that point.

This assumption is not always valid, and in fact there has been some confusion in the field about the relationship among least squares estimates, maximum likelihood estimates, and correlation when the underlying distribution is Gaussian. It is assumed they are the same. This is not true in general, and we recommend a thorough study of their relationships under various conditions.

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V.3 Non-Gaussian Correlation Models

We now describe a general stochastic model underlying and extending the framework of Mostafavi and Smith [78] and allowing mathematical formulation of one form of the problem of sub-pixel accuracy of image registration. We suppose that over a region of interest there exists a reference grey level field $Z_R(\underline{x})$ ($\underline{x}$ is a point (x,y) in the plane), including noise, which may be modelled as a (strictly) stationary random field, i.e., a collection of random variables all of whose joint distributions remain invariant under translation of the plane origin. The remotely sensed grey level field is modelled as the sum $Z_S(\underline{x}) = Z_D(\underline{x}) + N(\underline{x})$, where $Z_D(\underline{x})$ is a distorted form of $Z_R(\underline{x})$ (Mostafavi and Smith considered the case of affine "geometric" distortion $Z_D(\underline{x}) = Z_R(A\underline{x} + \underline{b})$ where A is a 2x2 matrix) and $N(\underline{x})$ is a stationary noise field assumed independent of $(Z_D(\underline{x}), Z_R(\underline{x}))$. We further assume that Z_D and Z_R are jointly stationary (have jointly translation-invariant statistics) and that the fields Z_D, Z_R, N have means 0 (i.e., means have been subtracted away) and known (or estimable) covariance functions

$$R_0(\underline{t}) = \mathrm{Cov}\{Z_R(\underline{x}), Z_R(\underline{x} + \underline{t})\} \text{ (for } \underline{x},\underline{t} \text{ in the plane)}$$

$$R_1(\underline{t}) = \mathrm{Cov}\{Z_D(\underline{x}), Z_D(\underline{x} + \underline{t})\}$$

$$R_{01}(\underline{t}) = \mathrm{Cov}\{Z_R(\underline{x}), Z_D(\underline{x} + \underline{t})\}$$

$$R_2(\underline{t}) = \mathrm{Cov}\{N(\underline{x}), N(\underline{x} + \underline{t})\}$$

(which do not depend on $\underline{x}$ because of stationarity).

Moreover, $Z_S(\cdot)$ is (due to the digital transmission of sensor images) for practical purposes only observed at lattice points $(x_0 + jh, y_0 + kh)$ in the plane, where (x_0, y_0) is an unknown offset from the origin of the reference image, h is the width of one pixel, and j and k are integers. In this setting the problem of image registration is to recover (estimate) (x_0, y_0) from the observations of $Z_R(\underline{x})$ over a large reference area and of $Z_S(\underline{x})$ over a much smaller area M in the plane. The statistics most often used for this estimation are the offset empirical

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correlations

$$\hat{C}_{j,k} \equiv \sum_{(l,m)\epsilon M_0} \frac{Z_S(lh,mh)Z_R((l + j)h,(m +k)h)}{\#(l,m) \text{ points in window } M_0}$$

where M_0 is a subset of M. (Mostafavi and Smith assume all fields Gaussian, which we do not, and do not deal with the discrete aspects of pixels).

There is extensive probability literature (see references) showing that for very large windows M (i.e., in the limit of infinite window size), sum-statistics of the form $\hat{C}_{j,k}$ considered as processes on the integer lattice in the plane are asymptotically Gaussian (as processes, and not simply as random variables) under various assumptions on the rate of decay of dependence among grey levels in groups of pixels which are widely separated. For this reason, our model problem is to recover (x_0,y_0) as best we can from knowledge of R_0,R_{01},R_1,R_2 and observation of $\hat{C}_{j,k}$ for all j,k in (some subset of) M_0. Note that we seek not simply to detect the "best" (j,k)-offset approximating (x_0,y_0) b, but to estimate (x_0,y_0) itself. Our criterion of performance will therefore be the probability that an estimated offset $(\hat{X},\hat{Y})$ differs from the true (x_0,y_0) by more than a distance (expressed for example as a fraction of a pixel width).

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CONCLUDING REMARKS

From the survey of published work on the analysis of registration accuracy, presented in the preceeding pages, it may be seen that little has been written on the analysis of subpixel registration accuracy. We recommend research be performed on the following aspects of subpixel accuracy:

- A study of the feasible registration accuracy attainable using binary edges for matching.
- A study of subpixel accuracy in images consisting of regions with linearly varying mean grey levels using correlation methods.
- An extension of the random field model of Mostafavi and Smith to the non-Gaussian subpixel case.

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